Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #5, October 11th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (6 pts.) find the derivatives of the following functions.

(a) $g(x) = \sec^4(3x) = [\sec(3x)]^4$

$g'(x) = 4[\sec(3x)]^3(\sec(3x)\tan(3x)) \cdot 3$

($\frac{d}{dx}(\sec(3x))$)

$= 12\sec^4(3x)\tan(3x)$.

(b) $f(x) = e^{x\cot x}$

$f'(x) = e^{x\cot x} \cdot \frac{d}{dx}[x\cot x]$

$= e^{x\cot x} \cdot (1\cdot\cot x + x\cdot(-\csc^2 x))$

$= (\cot x - x\csc^2 x)\, e^{x\cot x}$

Exercise 2. (6 pts.) Differentiate the following functions.

(a) $f(\theta) = \theta\sin\theta\cos\theta$

$f'(\theta) = 1\cdot(\sin\theta\cos\theta) + \theta\cdot\frac{d}{d\theta}(\sin\theta\cos\theta)$

$= \sin\theta\cos\theta + \theta\cdot[\sin\theta\cdot(-\sin\theta) + \cos\theta\cdot\cos\theta]$

$= \sin\theta\cos\theta + \theta(\cos^2\theta - \sin^2\theta)$

(b) $f(t) = 2^{3t^2} = (2)^{(3t^2)}$

$f'(t) = (\ln 2)(2^{3t^2})\cdot\frac{d}{dt}(3t^2)$

$= (\ln 2)\,2^{3t^2}\cdot 6t$

$= (6\ln 2)\,t\cdot 2^{3t^2}$

Exercise 3. (4 pts.) For what values of x does $y = \sqrt{x^2+3x}$ have a horizontal tangent?

$y = (x^2+3x)^{1/2}$

$y' = \frac{1}{2}(x^2+3x)^{-1/2}(2x+3)$

$= \frac{2x+3}{2\sqrt{x^2+3x}} = 0$

↑ because we want x values where slope is 0.

So we need

$2x+3 = 0$

So $x = -3/2$.

But, $x = -3/2$ is not in the domain of the derivative y'.

ans: y has no horizontal tangents

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Exercise 4. (4 pts.) Find an equation of the tangent line to the curve $y = \dfrac{6}{(\tan x + 3)^2}$ at the point $(0, 2)$

point

$y = 6(\tan x + 3)^{-2}$

$y' = -12(\tan x + 3)^{-3}(\sec^2 x)$

$= \dfrac{-12\sec^2 x}{(\tan x + 3)^3}$

$y'(0) = \dfrac{-12\sec^2 0}{(\tan 0 + 3)^3} = \dfrac{-12}{3^3} = \dfrac{-4}{9} = m$ ← slope

tangent line:

$y - 2 = -\frac{4}{9}(x - 0)$

$y = -\frac{4}{9}x + 2$

Correction:
The point should have been $(0, 2/3)$. So the line should have been $y = -\frac{4}{9}x + \frac{2}{3}$

Exercise 5. (5 pts.) Find the 50th derivative of $y = \sin(3x)$.

(a) Find the first four derivatives of $y = \sin(3x)$.

$y' = 3\cos(3x)$

$y'' = -3^2\sin(3x)$

$y''' = -3^3\cos(3x)$

$y^{(4)} = 3^4\sin(3x)$

(b) Using your answer to (a), find the 50th derivative of $y = \sin(3x)$.

$$\begin{array}{r} 12 \\ 4\overline{)50} \\ \underline{4} \\ 10 \\ \underline{8} \\ 2 \end{array}$$

← 2 derivatives past $4 \cdot 12 = 48$.

Every 4 derivatives, we get back to $\sin(3x)$.

Ans: $y^{(50)} = -3^{50}\sin(3x)$